

A
Project Report on

Synchronization Techniques for OFDM

In partial fulfillment of the requirements of
Bachelor of Technology (Electronics and
Communication Engineering)

Submitted By
Govind Singh Parihar (Roll No.10509016)
Session: 2008-09



**Department of Electronics and Communication
Engineering
National Institute of Technology
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Under the guidance of
Prof. Poonam Singh



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National Institute of Technology Rourkela

CERTIFICATE

This is to certify that that the work in this thesis report entitled “**Synchronization Techniques for OFDM**” submitted by Govind Singh Parihar in partial fulfillment of the requirements for the degree of Bachelor of Technology in Electronics and Communication Engineering Session 2005-2009 in the department of Electronics and Communication Engineering, National Institute of Technology Rourkela, Rourkela is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other University /Institute for the award of any degree.

Date:

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ACKNOWLEDGEMENT

I owe a debt of deepest gratitude to my thesis supervisor, **Ms. Poonam Singh**, Professor, Department of Electronics and Communication Engineering, for his guidance, support, motivation and encouragement through out the period this work was carried out. His readiness for consultation at all times, his educative comments, his concern and assistance even with practical things have been invaluable.

I want to thank all my teachers **Prof. G.S. Rath, Prof. K. K. Mahapatra, Prof. S.K. Patra** and **Prof. S.K. Meher** for providing a solid background for my studies and research thereafter. They have been great sources of inspiration to me and I thank them from the bottom of my heart.

I am grateful to **Prof. S.K.Patra**, Head of the Department, Electronics and Communication Engineering for providing us the necessary opportunities for the completion of our project. I also thank the other staff members of my department for their invaluable help and guidance.

I would like to thank all my friends and especially my classmates for all the thoughtful and mind stimulating discussions we had, which prompted us to think beyond the obvious. I've enjoyed their companionship so much during my stay at NIT, Rourkela.

I would like to thank all those who made my stay in Rourkela an unforgettable and rewarding experience.

Last but not least I would like to thank my parents, who taught me the value of hard work by their own example. They rendered me enormous support during the whole tenure of my stay in NIT Rourkela.

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B.Tech. Final Yr., ECE.

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ABSTRACT

FDMA, TDMA and CDMA are the well known multiplexing techniques used in wireless communication systems. While working with the wireless systems using these techniques various problems encountered are (1) multi-path fading (2) time dispersion which lead to intersymbol interference (ISI) (3) lower bit rate capacity (4) requirement of larger transmit power for high bit rate and (5) less spectral efficiency. In a typical terrestrial broadcasting, the transmitted signal arrives at the receiver using various paths of different lengths. Since multiple versions of the signal interfere with each other, it becomes difficult to extract the original information. The use of orthogonal frequency division multiplexing (OFDM) technique provides better solution for the above mentioned problems. OFDM technique distributes the data over a large number of carriers that are spaced apart at precise frequencies. This spacing provides the "orthogonality", which prevents the demodulator from seeing frequencies other than their own. The benefits of OFDM are high spectral efficiency, resiliency of RF interference, and lower multi-path distortion. OFDM is a powerful modulation technique that is capable of high data rate and is able to eliminate ISI. The use of FFT technique to implement modulation and demodulation functions makes it computationally more efficient. The OFDM based wireless communication system design includes the design of OFDM transmitter, and OFDM receiver. Using MATLAB, simulation of OFDM was done with different modulation techniques using different transform techniques. The digital modulation schemes such as BPSK and QPSK were

From the simulation results, it is observed that the BPSK allows the BER to be improved in a noisy channel at the cost of maximum data transmission capacity. Use of QPSK allows higher transmission capacity, but at the cost of slight increase in the probability of error. From the results, use of OFDM with QPSK is beneficial for short distance transmission link, whereas for long distance transmission link OFDM with BPSK will be preferable.

Maximum likelihood Estimation method is used for the prediction of timing and frequency offsets introduced by channel. It has been shown that ML estimation method improves the performance of the system very effectively. There are several other techniques also for prediction of timing and frequency offsets of an OFDM system., but in this paper ML is main area of consideration.

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1. INTRODUCTION

1.1 INTRODUCTION:

In a basic communication system, the data are modulated onto a single carrier frequency. The available bandwidth is then totally occupied by each symbol. This kind of system can lead to inter-symbol-interference (ISI) in case of frequency selective channel. The basic idea of OFDM is to divide the available spectrum into several orthogonal sub channels so that each narrowband sub channel experiences almost flat fading. Orthogonal frequency division multiplexing (OFDM) is becoming the chosen modulation technique for wireless communications. OFDM can provide large data rates with sufficient robustness to radio channel impairments. Many research centers in the world have specialized teams working in the optimization of OFDM systems. In an OFDM scheme, a large number of orthogonal, overlapping, narrow band sub-carriers are transmitted in parallel. These carriers divide the available transmission bandwidth. The separation of the sub-carriers is such that there is a very compact spectral utilization. With OFDM, it is possible to have overlapping sub channels in the frequency domain, thus increasing the transmission rate. The attraction of OFDM is mainly because of its way of handling the multipath interference at the receiver. Multipath phenomenon generates two effects (a) Frequency selective fading and (b) Intersymbol interference (ISI).

The "flatness" perceived by a narrowband channel overcomes the frequency selective fading. On the other hand, modulating symbols at a very low rate makes the symbols much longer than channel impulse response and hence reduces the ISI. Use of suitable error correcting codes provides more robustness against frequency selective fading. The insertion of an extra guard interval between consecutive OFDM symbols can reduce the effects of ISI even more. The use of FFT technique to implement modulation and demodulation functions makes it computationally more efficient. OFDM systems have gained an increased interest during the last years. It is used in the European digital broadcast radio system, as well as in wired environment such as asymmetric digital subscriber lines (ADSL). This technique is used in digital subscriber lines (DSL) to provides high bit rate over a twisted-pair of wires.

1.2 MOTIVATION:

Multimedia is effectively an infrastructure technology with widely different origins in computing, telecommunications, entertainment and publishing. New applications are emerging, not just in the wired environment, but also in the mobile one. At present, only low bit-rate data services are available to the mobile users. The radio environment is harsh, due to the many reflected waves and other effects. Using adaptive equalization techniques at the receiver could be the solution, but there are practical difficulties in operating this equalization in real-time at several Mb/s with compact, low-cost hardware. A promising candidate that eliminates a need for the complex equalizers is the Orthogonal Frequency Division Multiplexing (OFDM), a multiple carrier modulation technique. OFDM is robust in adverse channel conditions and allows a high level of spectral efficiency. It effectively mitigates performance degradations due to multipath and is capable of combating deep fades in part of the spectrum. The OFDM waveform can be easily modified to adjust to the delay spread of the channel. OFDM can handle large delay spreads easier to due the independence of the carriers and the flexibility of varying the cyclic prefix length. OFDM allows efficient operation in both FDD and TDD mode as very short or no pre-ambls are needed. Multiple access techniques which are quite developed for the single carrier modulations (e.g. TDMA, FDMA) had made possible of sharing one communication medium by multiple number of users. Multiple techniques schemes are used to allow many mobile users to share simultaneously a finite amount of radio spectrum. The sharing is required to achieve high capacity by simultaneously allocating the available bandwidth (or the available amount of channels) to multiple users.

For the quality communications, this must be done without severe degradation in the performance of the system. FDMA, TDMA and CDMA are the well known multiplexing techniques used in wireless communication systems. While working with the wireless systems using these techniques various problems encountered are (1) multi-path fading (2) time dispersion which lead to intersymbol interference (ISI) (3) lower bit rate capacity (4) requirement of larger transmit power for high bit rate and (5) less spectral efficiency. Disadvantage of FDMA technique is its Bad Spectrum Usage. Disadvantages of TDMA technique is Multipath Delay spread problem. In a typical terrestrial broadcasting, the transmitted signal arrives at the receiver using various paths of different lengths. Since multiple versions of the signal interfere with each other, it becomes difficult to extract the original information. The

use of orthogonal frequency division multiplexing (OFDM) technique provides better solution for the above mentioned problems.

1.3 LITERATURE SURVEY:

The concept of using parallel data transmission by means of frequency division multiplexing (FDM) was published in mid 60's [23, 24]. Some early development with this can be traced back to the 50s. A U.S. patent was filled and issued in January 1970. The idea was to use parallel data streams and FDM with overlapping sub channels to avoid the use of high-speed equalization and to combat impulsive noise, and multipath distortion as well as to fully use the available bandwidth. The initial applications were in the military communications. In the telecommunications field, the terms of discrete multi-tone (DMT), multichannel modulation and multicarrier modulation (MCM) are widely used and sometimes they are interchangeable with OFDM. In OFDM, each carrier is orthogonal to all other carriers. However, this condition is not always maintained in MCM. OFDM is an optimal version of multicarrier transmission schemes. Weinstein and Ebert [25] applied the discrete Fourier transform (DFT) [12] to parallel data transmission system as part of the modulation and demodulation process. In the 1980s, OFDM has been studied for high speed modems, digital mobile communications [10] and high-density recording.

Various fast modems were developed for telephone networks. In 1990s, OFDM has been exploited for wideband data communications over mobile radio FM channels [14], wireless LAN [13] wireless multimedia communication, high-bit-rate digital subscriber lines (HDSL) [16], asymmetric digital subscriber lines (ADSL) [20], very high speed digital subscriber lines (VHDSL), digital audio broadcasting (DAB) [18] and HDTV terrestrial broadcasting.

In a classical parallel data system, the total signal frequency band is divided into N nonoverlapping frequency subchannels. Each subchannel is modulated with a separate symbol and then the N sub channels are frequency-multiplexed. It seems good to avoid spectral overlap of channels to eliminate inter-channel interference. However, this leads to inefficient use of the available spectrum. To cope with the inefficiency, the ideas proposed from the mid-1960s were to use parallel data and FDM with overlapping sub channels, in which, each carrying a signaling

rate b is spaced b apart in frequency to avoid the use of high-speed equalization and to combat impulsive noise and multipath distortion, as well as to fully use the available bandwidth.

1.4 CONTRIBUTION:

Using MATLAB, simulation of OFDM was done with different modulation techniques using different transform techniques. The digital modulation schemes such as BPSK and QPSK were selected to assess the performance of the designed OFDM system by finding their Bit Error rate for different values of SNR.

There are various methods for the estimation of timing and frequency offsets for an OFDM system. Maximum likelihood method uses the statistical method of Maximum likelihood function for the estimation of the parameters. **Maximum likelihood estimation (MLE)** is a popular [statistical](#) method used for fitting a mathematical model to data. The modeling of real world data using estimation by maximum [likelihood](#) offers a way of tuning the free parameters of the model to provide a good fit.

Using MATLAB Maximum likelihood estimation of timing and frequency offsets for an OFDM system was simulated.

1.5 THESIS OUTLINE:

Following the introduction, the rest of the thesis is organized as follows. Chapter 2 gives a review on existing multiple access techniques used in wireless communication systems. Chapter 3 describes different digital modulation techniques that are used in OFDM.

Chapter 4 describes about different transform techniques that are available like FFT, DHT, and DCT. Chapter 5 gives an over view of OFDM. It describes OFDM basic principle, its working model, properties, parameters, and applications. In Chapter 6 describes OFDM simulations and results. Then I made a conclusion to my work and the points to possible directions for future work in Chapter 7.

2.MULTIPLE ACCESS TECHNIQUES

2.1 INTRODUCTION

Multiple access techniques are employed side by side in cellular systems. The need for multiple access techniques arises from the necessity to share a limited resource of radio spectrum amongst many users. Multiple access schemes are used to allow many mobile users to share simultaneously a finite amount of radio spectrum. The sharing of spectrum is required to achieve high capacity by simultaneously allocating the available bandwidth (or the available amount of channels) to multiple users.

2.2 DUPLEXING

In wireless communications systems, it is often desirable to allow the subscriber to send simultaneously while receiving information to the base station while receiving information from the base station. This effect is called *duplexing* and a device called duplexer is used inside each subscriber unit and base station. Duplexing may be done using frequency or time domain techniques. Figure 2.1 illustrates FDD (Frequency Division Duplexing) and TDD (Time Division Duplexing).

Frequency division duplexing (FDD) provides two distinct bands of frequencies for every user. The *forward band* provides traffic from the base station to mobile, and the reverse provides traffic from the mobile to base station.

Time division duplexing (TDD) uses time instead of frequency to provide both a forward and reverse link. In TDD, multiple users share a single radio channel by taking turns in the time domain. Individual users allowed to access the channel in assigned time slots, and each duplex channel has both forward time slot and a reverse time slot to facilitate bidirectional communication.

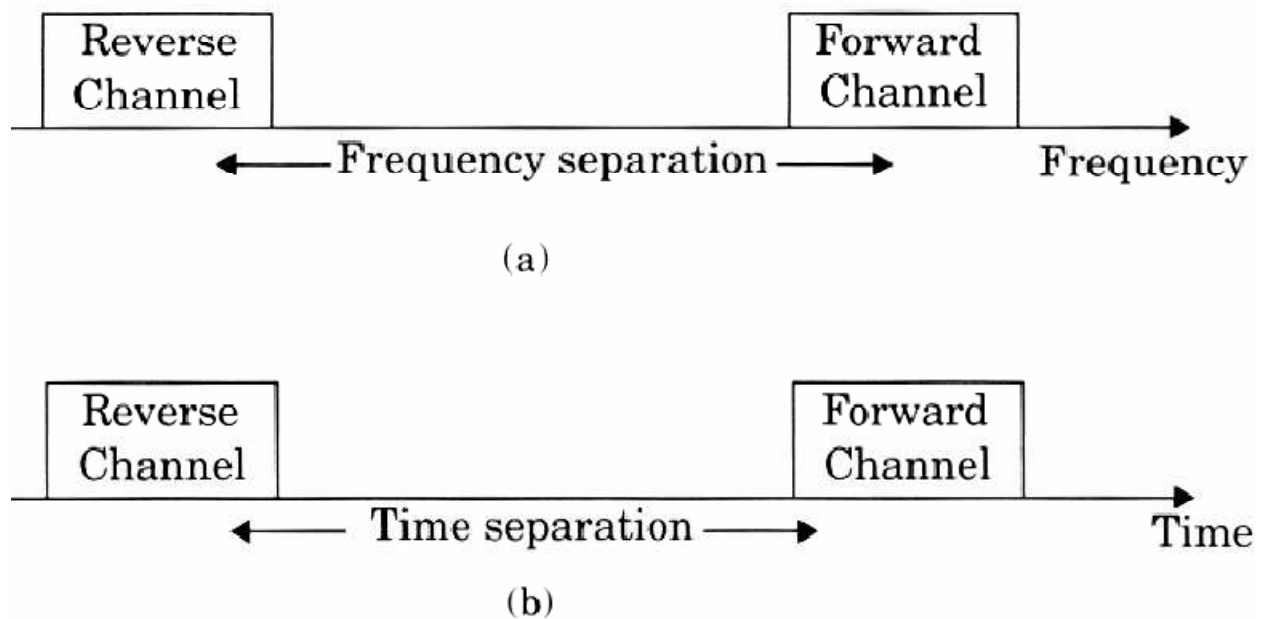


Figure 2.1: (a) FDD provides two simplex channels at the same time; (b) TDD provides two simplex time slots at the same frequency.

2.3 INTRODUCTION TO MULTIPLE ACCESS:

Frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA) are the three major access techniques used to share the available bandwidth in a wireless communication system

2.3.1 FDMA (Frequency Division Multiple Access): Frequency Division Multiple Access was the initial multiple-access technique for cellular systems. In this technique a user is assigned a pair of frequencies when placing or receiving a call. One frequency is used for downlink (base station to mobile) and one pair for uplink (mobile to base). This is called frequency division duplexing. That frequency pair is not used in the same cell or adjacent cells during the call. During the period of the call, no other user can share the same channel. If an FDMA channel is not in use, then it sits idle and cannot be used by other users to increase or share capacity. It is essentially a wasted resource. Figure 2.2 illustrates a FDMA system. Even though the user may not be talking, the spectrum cannot be reassigned as long as a call is in place. Frequency Division Multiple Access (FDMA) is the most common analog system. It is a technique whereby spectrum is divided up into frequencies and then assigned to users. With FDMA, only one

subscriber at any given time is assigned to a channel. The channel therefore is closed to other conversations until the initial call is finished, or until it is handed-off to a different channel. A “full-duplex” FDMA transmission requires two channels, one for transmitting and the other for receiving. FDMA has been used for first generation analog systems.

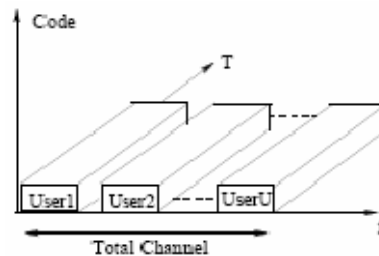


Figure 2.2: Frequency division multiple access (FDMA).

2.3.2 TDMA (*Time Division Multiple Access*): Time Division Multiple Access (TDMA)

improves spectrum capacity by splitting each frequency into time slots. TDMA allows each user to access the entire radio frequency channel for the short period of a call. Other users share this same frequency channel at different time slots. The base station continually switches from user to user on the channel. TDMA is the dominant technology for the second generation mobile cellular networks. TDMA system divide the radio spectrum into time slots, and in each slot only one user is allowed to transmit and receive. It can be seen from Figure 2.3 that each user occupies a cyclically repeating time slot, so a channel may be thought of as a particular time slot that reoccurs every frame, where N time slots comprise a frame.

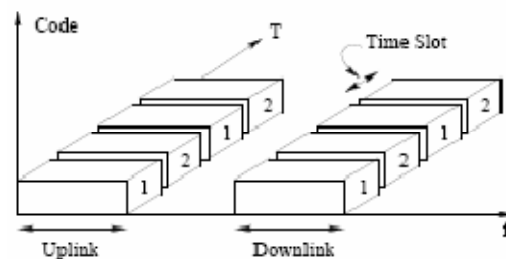


Figure 2.3: Time division multiple access (TDMA).

2.3.3 CDMA (*Code Division Multiple Access*): Code Division Multiple Access is based on “spread” spectrum technology. Since it is suitable for encrypted transmissions, it has long been used for military purposes. CDMA increases spectrum capacity by allowing all users to occupy

all channels at the same time. Transmissions are spread over the whole radio band, and each voice or data call are assigned a unique code to differentiate from the other calls carried over the same spectrum. CDMA allows for a “soft hand-off”, which means that terminals can communicate with several base stations at the same time. In CDMA systems, the narrowband message signal is multiplied by a very large bandwidth signal called the *spreading signal*. The spreading signal is a pseudo-noise code sequence that has a chip rate which is orders of magnitudes greater than the data rate of the message. All users in a CDMA system, as seen from figure 2.4, use the same carrier frequency and may transmit simultaneously. Each user has its own pseudorandom codeword which is approximately orthogonal to all other code words. The receiver performs a time correlation operation to detect only the specific desired codeword. All other code words appear as noise due to decorrelation. For detection of the message signal, the receiver needs to know the codeword used by the transmitter. Each user operates independently with no knowledge of the other users.

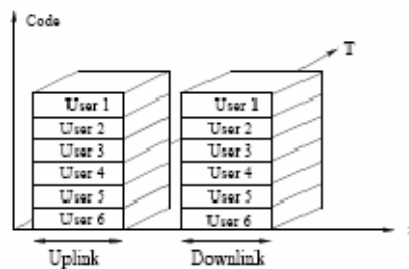


Figure 2.4: Code division multiple access (CDMA).

3. DIGITAL MODULATION TECHNIQUES

3.1 BASIC CONCEPTS OF MODULATION

3.1.1 Three kinds of modulations

Modulation is the process of facilitating the transfer of information over a medium. Sound transmission in air has limited range for the amount of power your lungs can generate. To extend the range your voice can reach, we need to transmit it through a medium other than air, such as a phone line or radio. The process of converting information (voice in this case) so that it can be successfully sent through a medium (wire or radio waves) is called modulation.

There are three basic types of digital modulation techniques. These are

1. Amplitude-Shift Keying (ASK)
2. Frequency-Shift Keying (FSK)
3. Phase-Shift Keying (PSK)

All of these techniques vary a parameter of a sinusoid to represent the information which we wish to send. A general carrier wave may be written:

$$C(t)=A\sin(2\pi ft+\Phi)$$

A sinusoid has three different parameters than can be varied. These are its amplitude, phase and frequency. Modulation is a process of mapping such that it takes your voice (as an example of a signal) converts it into some aspect of a sine wave and then transmits the sine wave, leaving the actual voice behind. The sine wave on the other side is remapped back to a near copy of your sound.

The medium is the thing through which the sine wave travels. So wire is a medium and so are air, water and space. The sine wave is called the carrier. The information to be sent, which can be voice or data is called the information signal. Once the carrier is mapped with the information to be sent, it is no longer a sine wave and we call it the signal.

3.2 ASK

In ASK, the amplitude of the carrier is changed in response to information and all else is kept fixed. In Binary ASK Bit 1 is transmitted by a carrier of one particular amplitude. To transmit 0, we change the amplitude keeping the frequency constant. On-Off Keying (OOK) is a special form of ASK, where one of the amplitudes is zero as shown in fig 3.1 and fig 3.2.

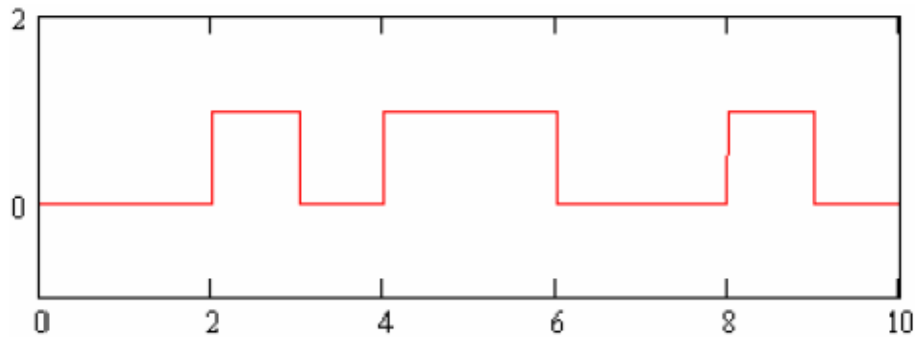


Figure 3.1 - Baseband information sequence – 0010110010

Binary ASK(t)=s(t)sin(2 π ft)

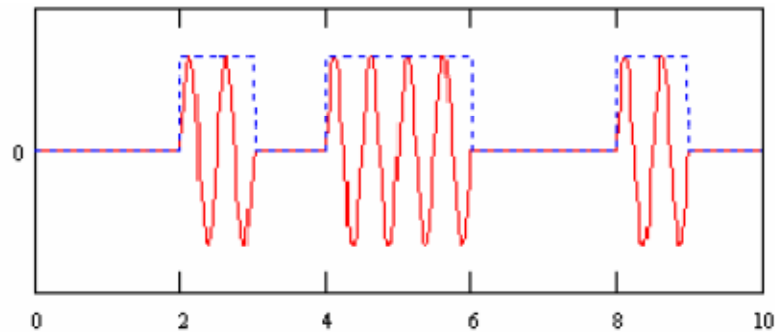


Figure 3.2 - Binary ASK (OOK) signal

3.3 FSK

In FSK, we change the frequency in response to information, In Binary FSK one particular frequency for a 1 and another frequency for a 0 is used as shown in fig 3.3 for the same bit sequence as above. In the example below, frequency f1 for bit 1 is higher than f2 used for the 0 bit.

$$\text{Binary FSK}(t) = \begin{cases} \sin(2\pi f_1 t) & \text{for bit 1} \\ \sin(2\pi f_2 t) & \text{for bit 0} \end{cases}$$

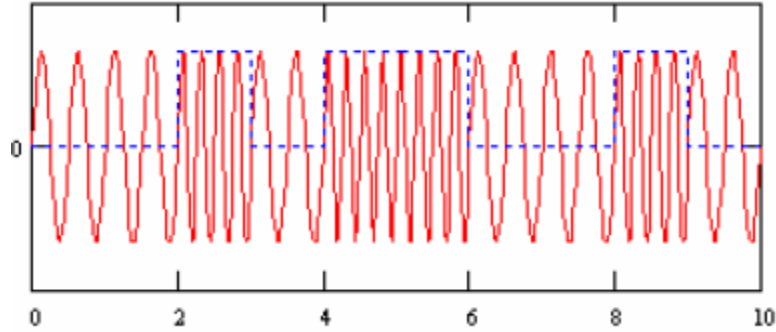


Figure 3.3 - Binary FSK signal

3.4 PSK

In PSK, we change the phase of the sinusoidal carrier to indicate information. Phase in this context is the starting angle at which the sinusoid starts. The transmitted signal is a sinusoid of fixed amplitude. Phase shift keying is a modulation process whereby the input signal, a binary PCM waveform, shifts the output waveform to one of a fixed number of states. The general analytic expression for PSK is

$$s_i(t) = \left(\frac{2E}{T}\right)^{1/2} \cos [\omega_0 + \phi_i(t)]$$

$0 \leq t \leq T$ $i=1, \dots, M$ Where the phase term $\phi_i(t)$ will have M discrete values, typically given by $\phi_i(t) = 2\pi i/M$ $i=1, \dots, M$ E is the symbol energy, T is symbol time duration.

For Binary PSK It has one fixed phase usually 0° when the data is 1. To transmit 0, we shift the phase of the sinusoid by 180° . Phase shift represents the change in the state of the information in this case. ASK techniques are most susceptible to the effects of non-linear devices which compress and distort signal amplitude. To avoid such distortion, the system must be operated in the linear range, away from the point of maximum power where most of the non-linear behavior occurs. The use of phase shift keying produces a constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading.

$$\text{Binary PSK}(t) = \begin{cases} \sin(2\pi ft) & \text{for bit 1} \\ \sin(2\pi ft + \pi) & \text{for bit 0} \end{cases}$$

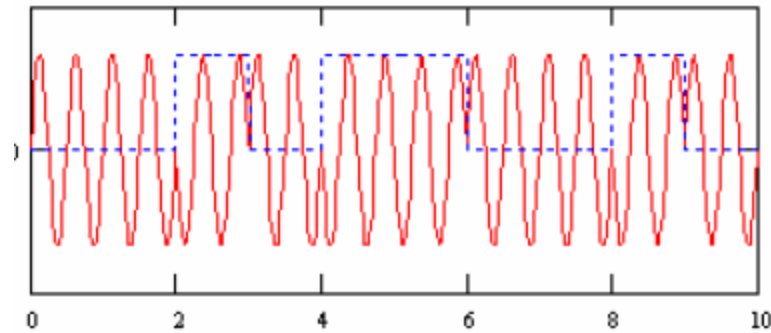


Figure 3.4 - Binary PSK Carrier (Note the 180° phase shifts at bit edges)

3.4.1 BPSK

In binary phase shift keying (BPSK) the transmitted signal is a sinusoid of fixed amplitude. BPSK is the simplest form of PSK. It uses two phases which are separated by 180° and so can also be termed 2-PSK. It has one fixed phase when the data is at one level and when the data is at another level the phase is different by 180°. It does not particularly matter exactly where the constellation points are positioned, and in this figure they are shown on the real axis, at 0° and 180°. This modulation is the most robust of all the PSK's, since it takes serious distortion to make the demodulator reach an incorrect decision. It is, however, only able to modulate at 1 bit/symbol (as seen in the figure 3.4) and so is unsuitable for high data-rate applications when bandwidth is limited.

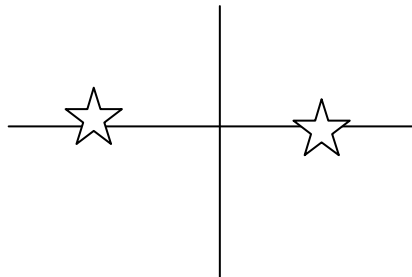


Fig 3.5 Constellation diagram for BPSK.

The bit error rate (BER) of BPSK in AWGN can be calculated as:

$$PP_b = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

Where $N_o/2$ is noise power spectral density (W/Hz)

Where T_b is bit interval

Where $E_b = P_s T_b$ is the energy contained in a bit duration

Where P_s is power of sinusoid of amplitude A.

So $P_s = 1/2A^2$

In binary phase-shift keying (BPSK) the transmitted signal is a sinusoid of fixed amplitude.

If the sinusoid is of amplitude A it has a power. $P_s = 1/2A^2$ and $A = \sqrt{P_s}$

Thus the transmitted signal is either

$$V_{BPSK}(t) = \sqrt{2P_s} \cos(\omega_o t)$$

$$V_{BPSK}(t) = \sqrt{2P_s} \cos(\omega_o t + \pi) = -\sqrt{2P_s} \cos(\omega_o t)$$

In BPSK the data $b(t)$ is a stream of binary digits with voltage levels which, as a matter of convenience, we take to be at +1v and -1v. When $b(t) = 1$ v we say it is at logic level 1 and when $b(t) = -1$ v we say it is at logic level 0. Hence V_{BPSK} can be written as

$$V_{BPSK}(t) = b(t) \sqrt{2P_s} \cos(\omega_o t)$$

So, the BPSK signal is generated as shown in the figure 3.6

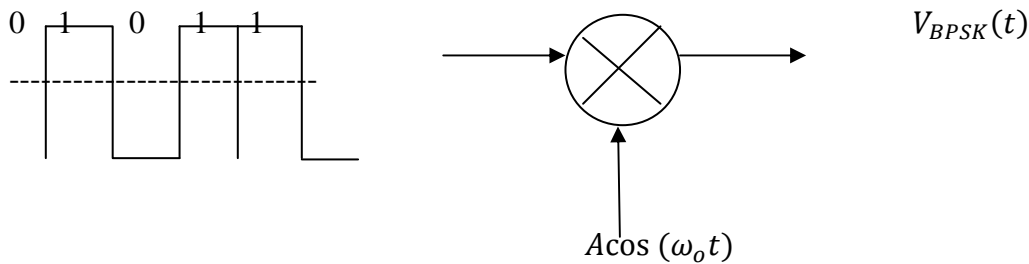


Figure 3.6: Illustrating of the modulation of a message signal to yield a BPSK signal.

In practice, a BPSK signal is generated by applying the waveform, $\cos(\omega_o t)$ as a carrier, to a balanced modulator and applying the baseband signal $b(t)$ as the modulating waveform. In this sense BPSK can be thought of as an AM signal.

If f_c is the frequency of the sinusoid and T is the bit interval then the spectrum of the resulting BPSK signal is shown in the figure 3.7

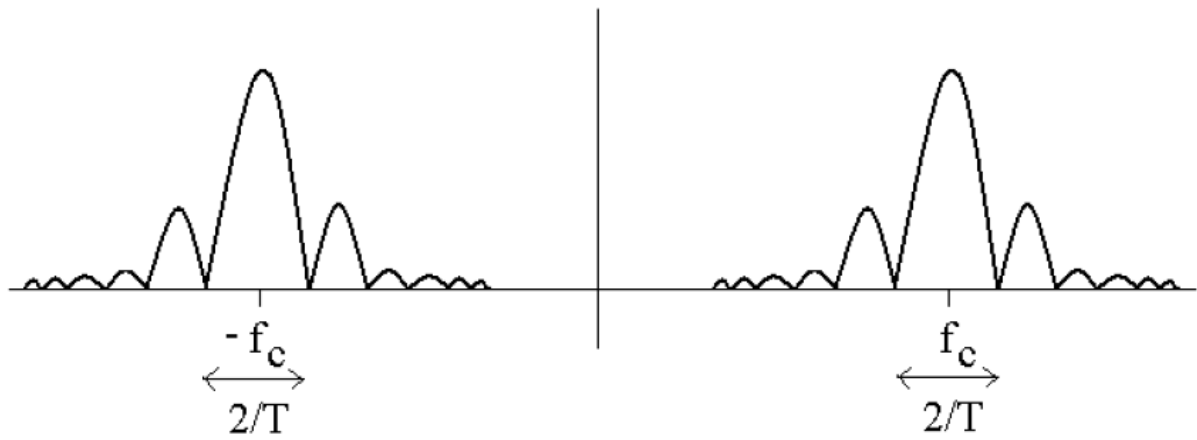


Figure 3.7: Amplitude spectrum of BPSK.

For binary sequence $m(t) = 0\ 1\ 0\ 1\ 0\ 0\ 1$, and if the sinusoid $s(t)$ is of amplitude of A , then the resulting BPSK signal will be as shown in the figure 3.8

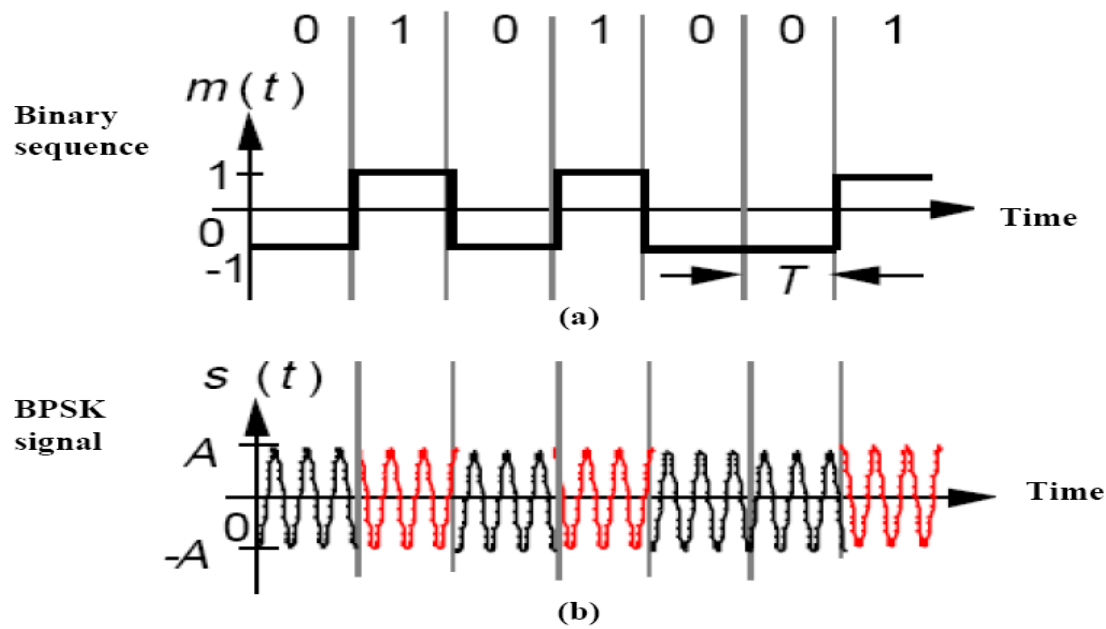


Figure 3.8 (a) Binary modulating signal, and (b) BPSK signal

3.4.2 QPSK

QPSK (4-ary PSK) involves changing the phase of the transmitted waveform. Each finite phase change represents unique digital data. A phase-modulated waveform can be generated by using the digital data to change the phase of a signal while its frequency and amplitude stay constant. A QPSK modulated carrier undergoes four distinct changes in phase that are represented as symbols and can take on the values of $\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$. Each symbol represents two binary bits of data. The constellation diagram of a QPSK modulated carrier is shown in Figure 3.9 $s(t) = x\cos(\omega t) + y\sin(\omega t)$

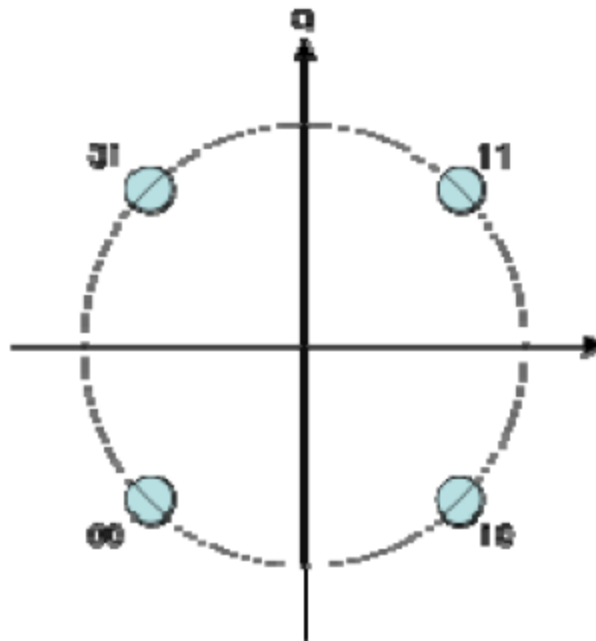


Figure 3.9: Constellation diagram for QPSK with Gray coding. Each adjacent symbol only differs by one bit.

3.4.2.1 Gray Code

The system performance of a digital communication network can be enhanced by incorporating a coding technique, within the system, known as Gray coding. The gray encoder is used to map the data in such a way as to help reduce bit errors. A QPSK system takes the input data bits, two at a time, and creates a symbol that represents one of four phase states. The gray encoder therefore is used to map every two input data bits to one of four unique symbol values so that the bit pairs that are used to generate the symbols are only one bit different from each adjacent symbol. This technique proves to help with error performance because if a symbol is received in error, it will contain only one error bit if it was received in error to an adjacent symbol. This can be more easily observed by viewing the QPSK constellation diagram shown in Fig 3.9. This QPSK constellation diagram shows symbols, each represented by two data bits that were first gray encoded. One can see that each adjacent symbol is represented by two data bits that vary by one bit. The performance of digital communication networks can further be enhanced by the use of error correcting codes.

3.4.2.2 QPSK Modulation

Figure 3.10 represents the process of a QPSK modulator. First, the input binary bit stream is split into two bit streams which are the even and odd bit streams (quadrature and in-phase streams) by the serial to parallel converter. Then, send alternating bits to I, Q channels: even bits to Q channel, odd bits to I channel.

Second, using the method of NRZ, the even and odd bits are converted from a unipolar sequence to a bipolar sequence (0 to -1). Next, multiply Q channel with a sine of f_c and multiply I channel with a sine but shifted by 90 degrees which is $-\cosine$. Notice that the 90 degrees block in the figure transmits the upper sine sequence to the lower $-\cosine$ sequence.

Finally, combining or adding the upper (I) and lower (Q) parts and passing through a harmonic or channel filter will get the QPSK modulated output.

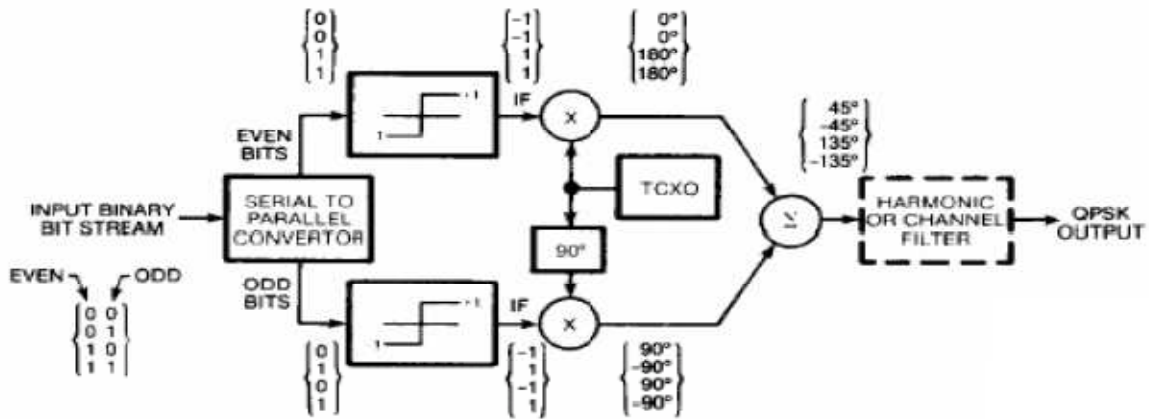


Figure 3.10: QPSK Modulator

For a binary sequence $m(t) = 00011011$, if the sinusoid $s(t)$ is of amplitude of A , then the resulting QPSK signal will be as shown in the figure 3.12. Phase of the sinusoid is shifted by 90° , 180° , 270° , 360° for data 00, 01, 10, 11 respectively

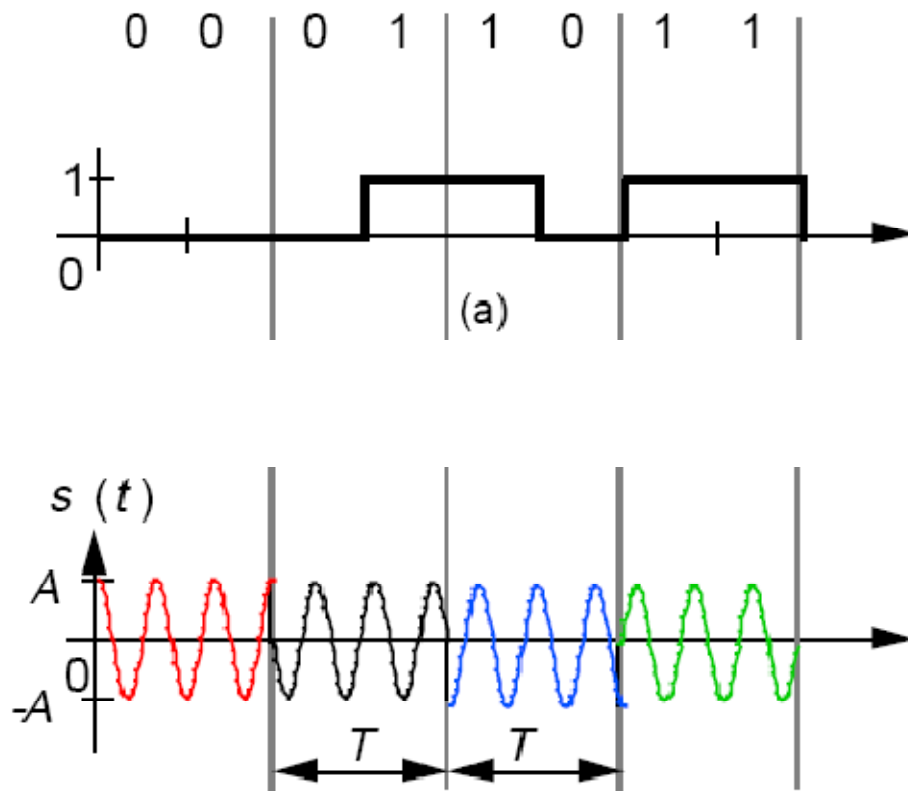


Figure 3.12 QPSK modulation: (a) binary sequence and (b) QPSK signal.

3.4.3 MPSK

In BPSK we transmit each bit individually. Depending on whether $b(t)$ is logic 0 or logic 1, we transmit one or another of sinusoid for the bit time T_b , the sinusoids differing in phase by $2\pi/2 = 180^\circ$. In QPSK we lump together two bits. Depending on which of the four two bits words develops, we transmit one or another of four sinusoids of duration $2T_b$, the sinusoids differing in phase by amount $2\pi/4 = 90^\circ$. The scheme can be extended. Let us lump together N bits so that in this N -bit symbol, extending over the time NT_b , there are $2^N = M$ possible symbols. Now let us represent the symbols by sinusoids of duration $N_b = T_s$ which differ from on another by the phase of $2\pi/M$.

Thus in M -ary PSK the waveforms used to identify the symbols are:

$$V_m(t) = \sqrt{2P_s} \cos(\omega_o + \phi_m) \quad (m=0,1,2,\dots,M-1)$$

With the symbol phase angel given by $\phi_m = \frac{(2m+1)\pi}{M}$

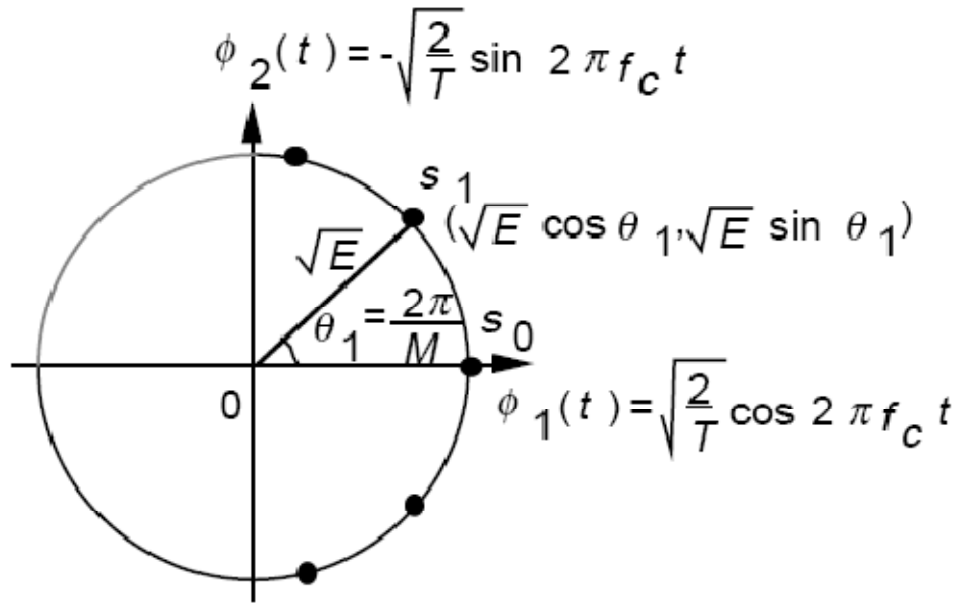


Figure 3.13: Constellation diagram for M -ary PSK

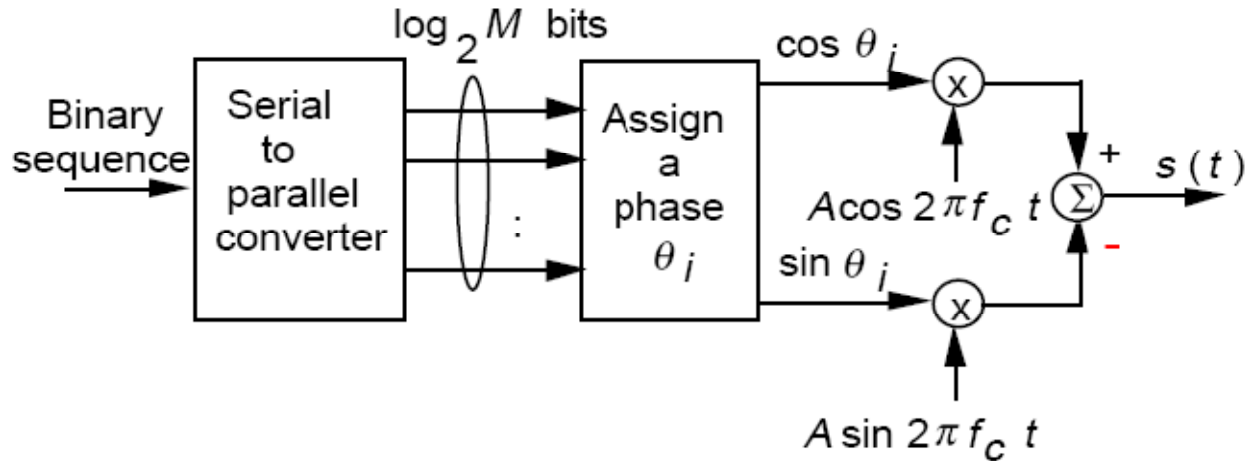


Figure 3.14 M-ary PSK modulator

3.5 QAM

In BPSK, QPSK, and M-ary PSK we transmit, in any symbol interval, one signal or another which are distinguished from one another in phase but are all of the same amplitude. In each of these individual systems the end points of the signal vectors in signal space falls on the circumference of a circle. Now we have note that our ability to distinguished one signal vector from another in the presence of noise will depend on the distance between the vector end points. It is hence rather apparent that we shall be able to improve the noise immunity of a system by allowing signal vectors to differ, not only in phase but also in amplitude. We call this as *amplitude and phase shift keying* or Quadrature amplitude modulation (QAM).

ASK is also combined with PSK to create hybrid systems such as Quadrature Amplitude Modulation (QAM) where both the amplitude and the phase are changed at the same time. QAM is a modulation scheme which conveys data by changing (*modulating*) the amplitude of two carrier waves. These two waves, usually sinusoids, are out of phase with each other by 90° and are thus called quadrature carriers—hence the name of the scheme.

As for many digital modulation schemes, the constellation diagram is a useful representation. In QAM, the constellation points are usually arranged in a square grid with equal vertical and horizontal spacing, although other configurations are possible. Since in digital telecommunications the data is usually binary, the number of points in the grid is usually a power of 2 (2, 4, 8...). Since QAM is usually square, some of these are rare—the most common forms

are 16-QAM, 64-QAM, 128-QAM and 256-QAM. By moving to a higher-order constellation, it is possible to transmit more bits per symbol. However, if the mean energy of the constellation is to remain the same (by way of making a fair comparison), the points must be closer together and are thus more susceptible to noise and other corruption; this results in a higher bit error rate and so higher-order QAM can deliver more data less reliably than lower-order QAM.

If data-rates beyond those offered by 8-PSK are required, it is more usual to move to QAM since it achieves a greater distance between adjacent points in the I-Q plane by distributing the points more evenly. The complicating factor is that the points are no longer all the same amplitude and so the demodulator must now correctly detect both phase and amplitude, rather than just phase.

64-QAM and 256-QAM are often used in digital cable television and cable modem applications. In the US, 64-QAM and 256-QAM are the mandated modulation schemes for digital cable. In the UK, 16-QAM and 64-QAM are currently used for digital terrestrial television (Freeview and Top Up TV).

3.5.1 Rectangular QAM

Rectangular QAM constellations are, in general, sub-optimal in the sense that they do not maximally space the constellation points for a given energy. However, they have the considerable advantage that they may be easily transmitted as two pulse amplitude modulation (PAM) signals on quadrature carriers, and can be easily demodulated. The non-square constellations achieve marginally better bit-error rate (BER) but are harder to modulate and demodulate.

The first rectangular QAM constellation usually encountered is 16-QAM, its constellation diagram is shown in fig. 3.15. A Gray coded bit-assignment is also given. The reason that 16-QAM is usually the first is that a brief consideration reveals that 2-QAM and 4-QAM are in fact binary phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK), respectively. Also, the error-rate performance of 8-QAM is close to that of 16-QAM (only about 0.5dB better), but its data rate is only three-quarters that of 16-QAM.

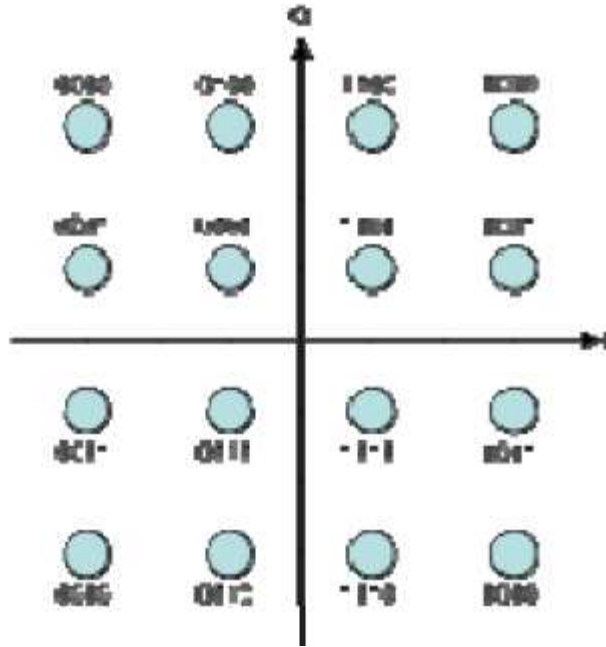


Figure 3.15: Constellation diagram for rectangular 16-QAM.

Expressions for the symbol error-rate of rectangular QAM are not hard to derive but yield rather unpleasant expressions. For an even number of bits per symbol, k , exact expressions are available. They are most easily expressed in a *per carrier* sense:

$$P_{sc} = 2 \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1} \frac{E_s}{N_o}}\right)$$

$$\text{So, } P_{sc} = 1 - (1 - P_{sc})^2$$

The bit-error rate will depend on the exact assignment of bits to symbols, but for a Gray-coded assignment with equal bits per carrier:

$$P_{bc} = 4/k \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1} \frac{E_s}{N_o}}\right)$$

$$P_b = 1 - (1 - P_{bc})^2$$

M = Number of symbols in modulation constellation

E_b = Energy-per-bit

E_s = Energy-per-symbol = kE_b with k bits per symbol

N_o = Noise power spectral density (W/Hz)

P_b = Probability of bit-error

P_{bc} = Probability of bit-error per carrier

P_s = Probability of symbol-error

P_{sc} = Probability of symbol-error per carrier

4. OFDM AND ML ESTIMATION

4.1 EVOLUTION OF OFDM

Frequency Division Multiplexing (FDM)

Frequency Division Multiplexing (FDM) has been used for a long time to carry more than one signal over a telephone line. FDM divides the channel bandwidth into subchannels and transmits multiple relatively low rate signals by carrying each signal on a separate carrier frequency. To ensure that the signal of one subchannel did not overlap with the signal from an adjacent one, some guard-band was left between the different subchannels. Obviously, this guard-band led to inefficiencies.

Orthogonal Frequency Division Multiplexing (OFDM)

In order to solve the bandwidth efficiency problem, orthogonal frequency division multiplexing was proposed, where the different carriers are orthogonal to each other. With OFDM, it is possible to have overlapping sub channels in the frequency domain, thus increasing the transmission rate. This carrier spacing provides optimal spectral efficiency. Today, OFDM has grown to be the most popular communication system in high-speed communications. OFDM is becoming the chosen modulation technique for wireless communications. OFDM can provide large data rates with sufficient robustness to radio channel impairments.

4.2 INTRODUCTION TO OFDM

Orthogonal Frequency Division Multiplexing (OFDM)

Modulation - *a mapping of the information on changes in the carrier phase, frequency or amplitude or combination.*

Multiplexing - *method of sharing a bandwidth with other independent data channels.*

OFDM is a combination of modulation and multiplexing. Multiplexing generally refers to independent signals, those produced by different sources. In OFDM the question of multiplexing is applied to independent signals but these independent signals are a sub-set of the one main

signal. In OFDM the signal itself is first split into independent channels, modulated by data and then re-multiplexed to create the OFDM carrier.

OFDM is a special case of Frequency Division Multiplex (FDM). In an OFDM scheme, a large number of orthogonal, overlapping, narrow band sub-carriers are transmitted in parallel. These carriers divide the available transmission bandwidth. The separation of the sub-carriers is such that there is a very compact spectral utilization.

As an analogy, a FDM channel is like water flow out of a faucet, a whole bunch of water coming all in one stream; In contrast the OFDM signal is like a shower from which same amount of water will come as a lot of small streams. In a faucet all water comes in one big stream and cannot be sub-divided. OFDM shower is made up of a lot of little streams.



Fig 4.1 (a) A Regular-FDM single carrier (b) Orthogonal-FDM

The advantage one over the other is that if I put my thumb over the faucet hole, I can stop the water flow but I cannot do the same for the shower. So although both do the same thing, they respond differently to interference. Both methods carry the exact same amount of data. But in case of any interfere to some of these small streams, only some part of data in the OFDM method will suffer.

These small streams when seen as signals are called the sub-carriers in an OFDM system and they must be orthogonal for this idea to work. The independent sub-channels can be multiplexed by frequency division multiplexing (FDM), called multi-carrier transmission or it can be based on a code division multiplex (CDM), in this case it is called multi-code transmission.

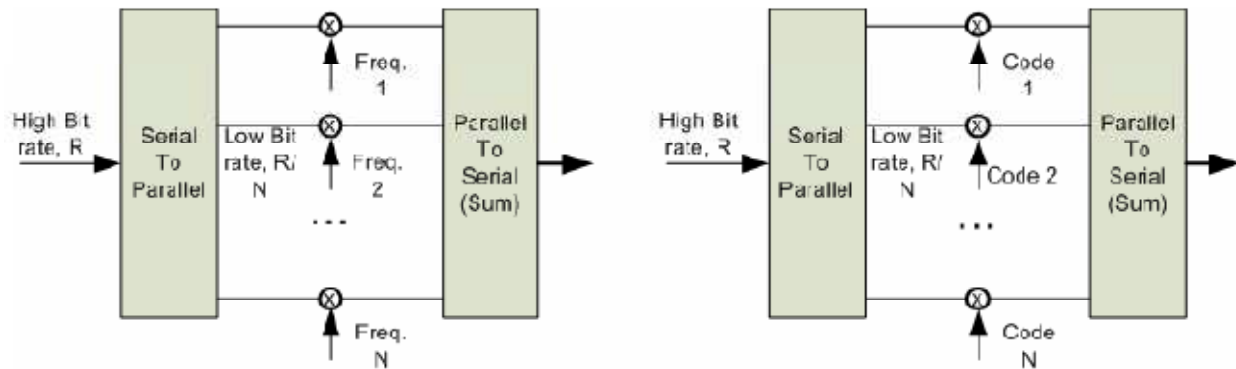


Fig 4.2 Multi-carrier FDM and Multi-code division multiplex

4.3 IMPORTANCE OF ORTHOGONALITY

The main concept in OFDM is orthogonality of the sub-carriers. The "orthogonal" part of the OFDM name indicates that there is a precise mathematical relationship between the frequencies of the carriers in the system. It is possible to arrange the carriers in an OFDM Signal so that the sidebands of the individual carriers overlap and the signals can still be received without adjacent carriers interference. In order to do this the carriers must be mathematically orthogonal. The Carriers are linearly independent (i.e. orthogonal) if the carrier spacing is a multiple of $1/T_s$. Where, T_s is the symbol duration. The orthogonality among the carriers can be maintained if the OFDM signal is defined by using Fourier transform procedures. The OFDM system transmits a large number of narrowband carriers, which are closely spaced. Note that at the central frequency of the each sub channel there is no crosstalk from other sub channels

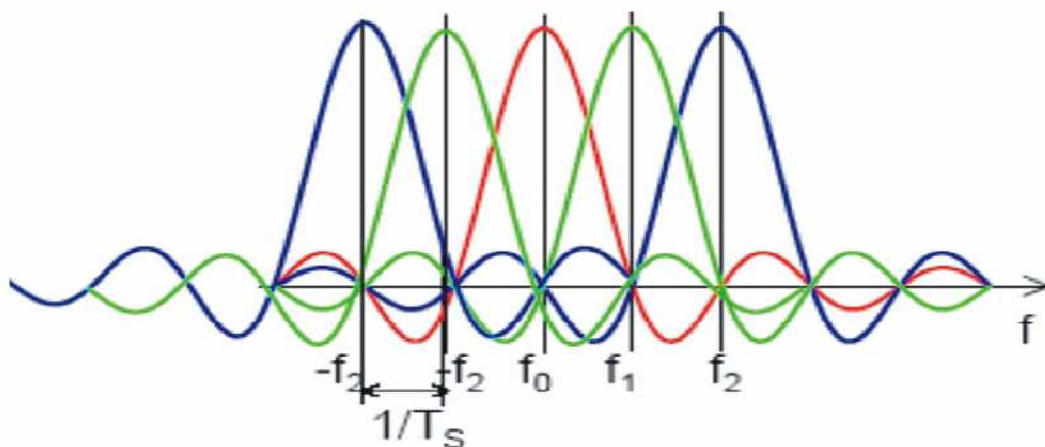


Fig 4.3 Example of OFDM spectrum for 5 orthogonal carriers

Since the carriers are all sine/cosine wave, we know that area under one period of a sine or a cosine wave is zero. This is easily shown.

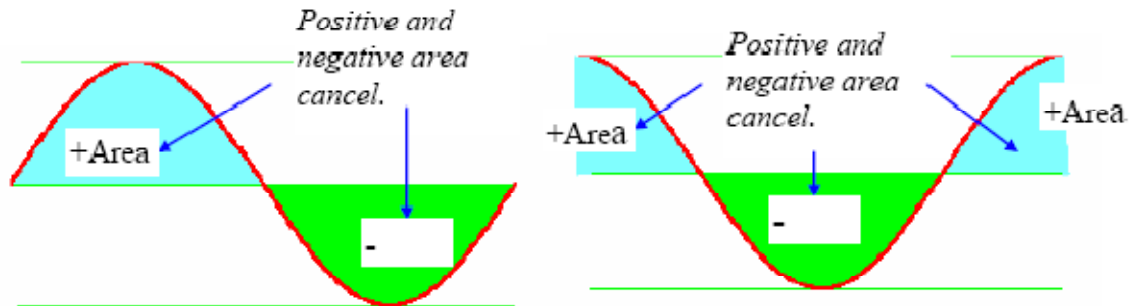


Fig. 4.4 - The area under a sine and a cosine wave over one period is always zero.

If a sine wave of frequency m multiplied by a sinusoid (sine or cosine) of a frequency n ,

$$f(t) = \sin(m\omega t) * \sin(n\omega t)$$

Where both m and n are integers, since these two components are each a sinusoid, the integral is equal to zero over one period. The integral or area under this product is given by

$$= \int_0^{2\pi} \frac{1}{2} \cos(m-n)\omega t - \int_0^{2\pi} \frac{1}{2} \cos(m+n)\omega t = 0 - 0$$

So when a sinusoid of frequency n multiplied by a sinusoid of frequency m/n , the area under the product is zero. In general for all integers n and m , $\sin mx$, $\cos mx$, $\cos nx$, $\sin nx$ are all orthogonal to each other. These frequencies are called harmonics. $f(t) = \sin \omega t * \sin n\omega t$

Sine wave multiplied by another of a different harmonic.

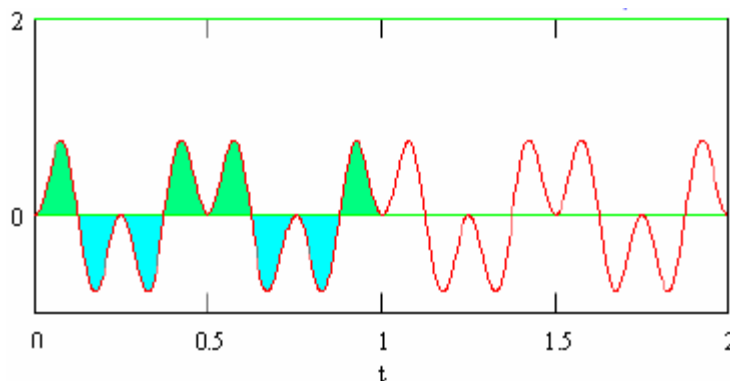


Fig 4.5 the area under a sine wave multiplied by its own harmonic is always zero

The orthogonality allows simultaneous transmission on a lot of sub-carriers in a tight frequency space without interference from each other. In essence this is similar to CDMA, where codes are used to make data sequences independent (also orthogonal) which allows many independent users to transmit in same space successfully.

4.4 OFDM is a special case of FDM

Frequency Division Multiplexing FDM is a special case of FDM. If I have a bandwidth that goes from frequency say a to b , I can subdivide this into a frequency of equal spaces. In frequency space the modulated carriers would look like this.



Fig. 4.6 – bandwidth utilization of FDM

The frequencies a and b can be anything, integer or non-integer since no relationship is implied between a and b . same is true of the carrier center frequencies which are based on frequencies that do not have any special relationship to each other. If frequency C_1 and C_n were such that for any n , an integer, the following holds.

$$C_n = n * c_1$$

So that

$$C_2 = 2c_1$$

$$C_3 = 3c_1$$

$$C_4 = 4c_1$$

All three of these frequencies are harmonic to c_1 .

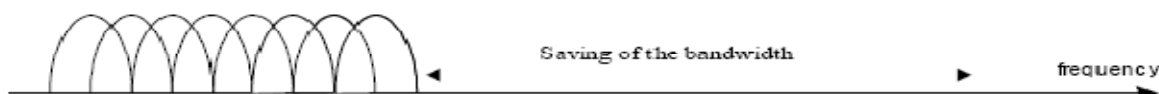


Fig 4.7 bandwidth utilization of OFDM

In this case, since these carriers are orthogonal to each other, when added together, they do not interfere with each other. In FDM, since we do not generally have frequencies that follow the above relationship, we get interference from neighbor carriers. To provide adjacent channel interference protection, signals are moved further apart. Each carrier may be placed apart allowing for a 10% guard band. The frequencies would not be orthogonal but in FDM we don't care about this. It's the guard band that helps keep interference under control.

4.5 Maximum Likelihood Estimation of Timing and Frequency Offset in OFDM.

1. The OFDM system model

The diagram below shows the basic data flow chart of an OFDM system

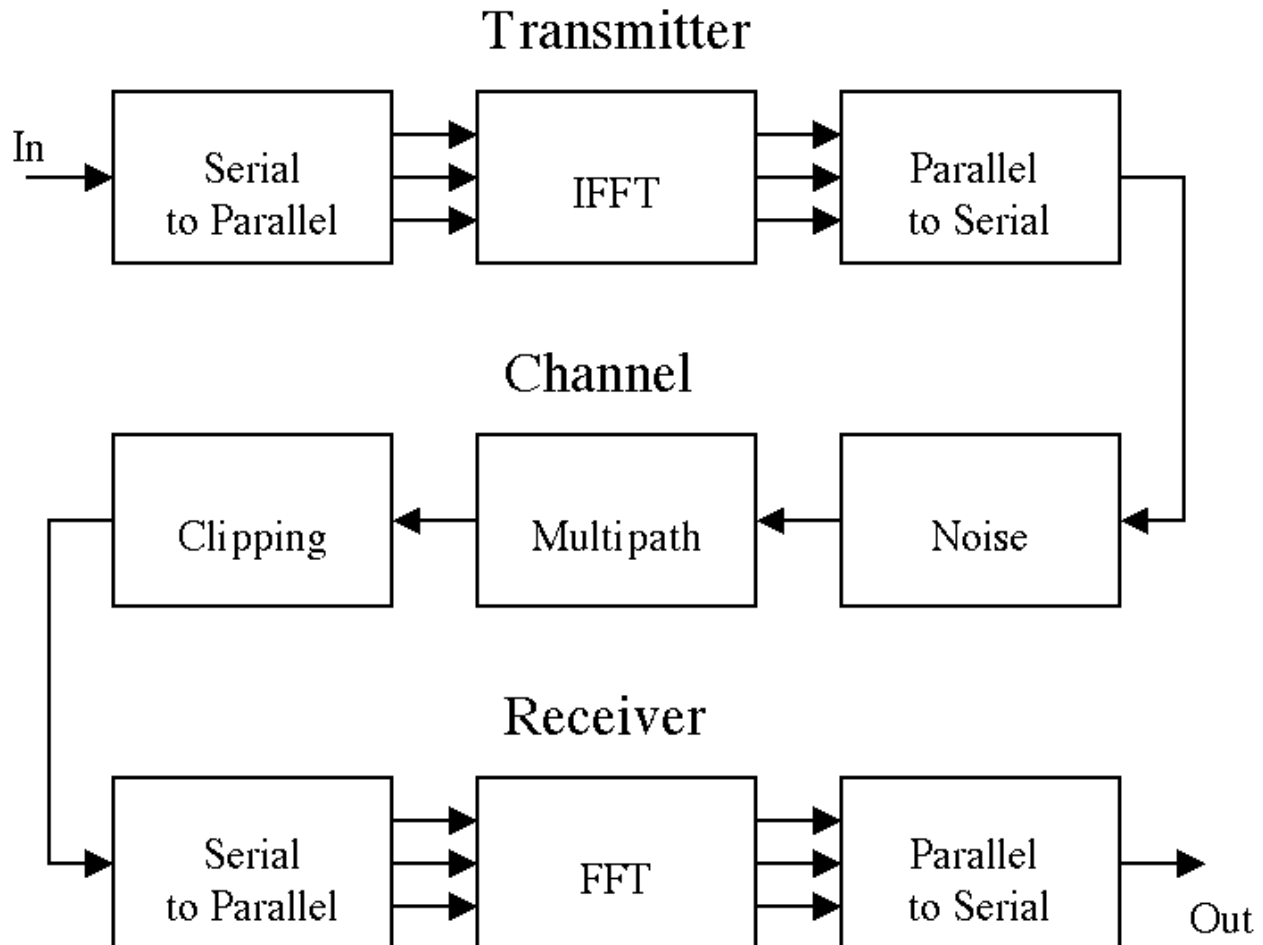


Fig 4.8 OFDM model

Fig. 1 illustrates the baseband, discrete-time OFDM system model we investigate. The complex data symbols are modulated by means of an *inverse discrete Fourier transform* (IDFT) on N -parallel subcarriers. The resulting OFDM symbol is serially transmitted over a discrete-time channel, whose impulse response we assume is shorter than L samples. At the receiver, the data are retrieved by means of a *discrete Fourier transform* (DFT). An accepted means of avoiding intersymbol interference (ISI) and preserving orthogonality between subcarriers is to copy the last L samples of the body of the OFDM symbol (N samples long) and append them as a preamble—the cyclic prefix—to form the complete OFDM symbol. The effective length of the OFDM symbol as transmitted is this cyclic prefix plus the body ($L+N$ samples long). The insertion of a cyclic prefix can be shown to result in an equivalent parallel orthogonal channel structure that allows for simple channel estimation and equalization. In spite of the loss of transmission power and bandwidth associated with the cyclic prefix, these properties generally motivate its use. In the following analysis, we assume that the channel is non dispersive and that the transmitted signal $s(k)$ is only affected by complex additive white Gaussian noise (AWGN) $n(k)$. We will, however, evaluate our estimator's performance for both the AWGN channel and a time-dispersive channel. Consider two uncertainties in the receiver of this OFDM symbol: the uncertainty in the arrival time of the OFDM symbol (such ambiguity gives rise to a rotation of the data symbols) and the uncertainty in carrier frequency (a difference in the local oscillators in the transmitter and receiver gives rise to a shift of all the subcarriers). The first uncertainty is modeled as a delay in the channel impulse response $\delta(k - \theta)$, where θ is the integer-valued unknown arrival time of a symbol.

The latter is modeled as a complex multiplicative distortion of the received data in the time domain $e^{j2\pi\epsilon k/N}$, where ϵ denotes the difference in the transmitter and receiver oscillators as a fraction of the intercarrier spacing ($1/N$ in normalized frequency). Notice that all subcarriers experience the same shift ϵ . These two uncertainties and the AWGN thus yield the received signal

$$r(k) = s(k - \theta)e^{j2\pi\epsilon k/N} + n(k)$$

Two other synchronization parameters are not accounted for in this model. First, an offset in the carrier phase may affect the symbol error rate in coherent modulation. If the data is differentially encoded, however, this effect is eliminated. An offset in the sampling frequency will also affect the system performance. We assume that such an offset is negligible. Now, consider the transmitted signal $s(k)$. This is the DFT of the data symbols X_k , which we assume are independent. Hence $s(k)$, is a linear combination of independent, identically distributed random variables. If the number of subcarriers is sufficiently large, we know from the central limit theorem that $s(k)$ approximates a complex Gaussian process whose real and imaginary parts are independent. This process, however, is not white since the appearance of a cyclic prefix yields a correlation between some pairs of samples that are spaced samples apart. Hence $r(k)$, is not a white process either, but because of its probabilistic structure, it contains information about the time offset θ and carrier frequency offset ϵ . This is the crucial observation that offers the opportunity for joint estimation of these parameters based on $r(k)$.

A synchronizer cannot distinguish between phase shifts introduced by the channel and those introduced by symbol time delays [4]. Time error requirements may range from the order of one sample (wireless applications, where the channel phase is tracked and corrected by the channel equalizer) to a fraction of a sample (in, e.g., high bit-rate digital subscriber lines, where the channel is static and essentially estimated only during startup). Without a frequency offset, the frequency response of each subchannel is zero at all other subcarrier frequencies, i.e., the subchannels do not interfere with one other [2]. The effect of a frequency offset is a loss of orthogonality between the tones. The resulting intercarrier interference (ICI) has been investigated in [11]. The effective signal-to-noise ratio SNR due to both additive noise and ICI is shown to be lower bounded by

$$SNR_e(\epsilon) \geq \frac{SNR}{1 + 0.5947 SNR \sin^2(\pi\epsilon)} \left(\frac{\sin \pi\epsilon}{\pi\epsilon} \right)^2$$

The difference between the SNR and the SNR_e is a measure of the sensitivity to a frequency offset. Notice that in the absence of additive noise, the frequency offset must satisfy $|\epsilon| \leq 0.13$ in order to obtain an SNR of 30 dB or higher. This result agrees well with the analysis of

multiuser OFDM systems in [3], which states that a frequency accuracy of 1–2% of the inter-carrier spacing is necessary.

2. ML estimation:

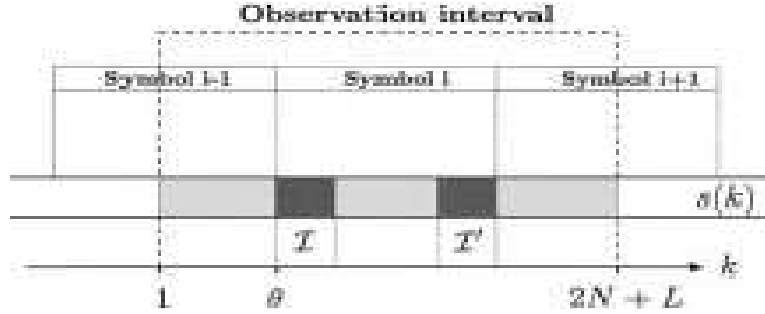


Fig. 4.9 Structure of OFDM signal with cyclically extended symbols $s(k)$: The set \mathcal{I} contains the cyclic prefix, i.e., the copies of the L data samples in \mathcal{I}'

Assume that we observe consecutive samples of, cf. Fig.4.9, and that these samples contain one complete-sample OFDM symbol. The position of this symbol within the observed block of samples, however, is unknown because the channel delay is unknown to the receiver. Define the index sets.

$$\mathcal{I} \triangleq \{\theta, \dots, \theta + L - 1\} \quad \text{and} \\ \mathcal{I}' \triangleq \{\theta + N, \dots, \theta + N + L - 1\}$$

The set \mathcal{I}' thus contains the indices of data samples that are copied into the cyclic prefix, and the set \mathcal{I} contains the indices of the prefix. collect the observed samples in the $(2N+L) \times 1$ -vector $\mathbf{r} \triangleq [r(1) \dots r(2N+L)]^T$. Notice that the samples in the cyclic prefix and their copies $r(k), k \in \mathcal{I} \cup \mathcal{I}'$ are pairwise correlated, i.e

$$\forall k \in \mathcal{I}: \quad E\{r(k)r^*(k+m)\} = \begin{cases} \sigma_s^2 + \sigma_n^2 & m = 0 \\ \sigma_s^2 e^{-j2\pi\epsilon} & m = N \\ 0 & \text{otherwise} \end{cases}$$

while the remaining samples $r(k), k \notin \mathcal{I} \cup \mathcal{I}'$ are mutually uncorrelated. The log-likelihood function for θ and ϵ , $\Lambda(\theta, \epsilon)$ is the logarithm of the probability density function $\mathcal{F}(\mathbf{r}|\theta, \epsilon)$ of

the $2N+L$ observed samples in \mathbf{r} given the arrival time θ and the carrier frequency offset ε . In the following, we will drop all additive and positive multiplicative constants that show up in the expression of the log-likelihood function since they do not affect the maximizing argument. Moreover, we drop the conditioning on (θ, ε) for notational clarity. Using the correlation properties of the observations \mathbf{r} , the log-likelihood function can be written as

$$\begin{aligned}\Lambda(\theta, \varepsilon) &= \log f(\mathbf{r}|\theta, \varepsilon) \\ &= \log \left(\prod_{k \in \mathcal{I}} f(r(k), r(k+N)) \prod_{k \notin \mathcal{I} \cup \mathcal{I}'} f(r(k)) \right) \\ &= \log \left(\prod_{k \in \mathcal{I}} \frac{f(r(k), r(k+N))}{f(r(k))f(r(k+N))} \prod_k f(r(k)) \right)\end{aligned}\tag{4}$$

where $f(\cdot)$ denotes the probability density function of the variables in its argument. Notice that it is used for both one- and two-dimensional (1-D and 2-D) distributions. The product in (4) is independent of θ (since the product is over all k) and ε (since the density $f(r(k))$ is rotationally invariant). Since the ML estimation of θ and ε is the argument maximizing $\Lambda(\theta, \varepsilon)$, we may omit this factor. Under the assumption that \mathbf{r} is a jointly Gaussian vector, (4) is shown in the Appendix to be

$$\Lambda(\theta, \varepsilon) = |\gamma(\theta)| \cos(2\pi\varepsilon + \angle\gamma(\theta)) - \rho\phi(\theta) \tag{5}$$

where \angle denotes the argument of a complex number.

$$\begin{aligned}\gamma(m) &\triangleq \sum_{k=m}^{m+L-1} r(k)r^*(k+N), \\ \Phi(m) &\triangleq \frac{1}{2} \sum_{k=m}^{m+L-1} |r(k)|^2 + |r(k+N)|^2\end{aligned}\tag{6,7}$$

and

$$\rho \triangleq \left| \frac{E\{r(k)r^*(k+N)\}}{\sqrt{E\{|r(k)|^2}E\{|r(k+N)|^2\}}} \right| = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} = \frac{SNR}{SNR + 1}$$

is the magnitude of the correlation coefficient between $\mathbf{r}(\mathbf{k})$ and $\mathbf{r}(\mathbf{k}+\mathbf{N})$. The first term in (5) is the weighted magnitude of $\gamma(m)$, which is a sum of L consecutive correlations between pairs of samples spaced N samples apart. The weighting factor depends on the frequency offset. The term $\phi(m)$ is an energy term, independent of the frequency offset ε . Notice that its contribution depends on the SNR (by the weighting-factor ρ).

The maximization of the log-likelihood function can be performed in two steps:

$$\max_{(\theta, \varepsilon)} \Lambda(\theta, \varepsilon) = \max_{\theta} \max_{\varepsilon} \Lambda(\theta, \varepsilon) = \max_{\theta} \Lambda(\theta, \hat{\varepsilon}_{\text{ML}}(\theta)).$$

The maximum with respect to the frequency offset ε is obtained when the cosine term in (5) equals one. This yields the ML estimation of ε

$$\hat{\varepsilon}_{\text{ML}}(\theta) = -\frac{1}{2\pi} \angle \gamma(\theta) + n$$

where n is an integer. A similar frequency offset estimator has been derived in [11] under different assumptions. Notice that by the periodicity of the cosine function, several maxima are found. We assume that an acquisition, or rough estimate, of the frequency offset has been performed and that $|\varepsilon| \leq 1/2$; thus $n=0$. Since $\cos(2\pi\varepsilon + \angle\gamma(\theta))=1$, the log-likelihood function of θ (which is the compressed log-likelihood function with respect to ε) becomes

$$\Lambda(\theta, \hat{\varepsilon}_{\text{ML}}(\theta)) = |\gamma(\theta)| - \rho\Phi(\theta)$$

and the joint ML estimation of θ and ε becomes

$$\hat{\theta}_{\text{ML}} = \arg \max_{\theta} \{|\gamma(\theta)| - \rho\phi(\theta)\}$$

$$\hat{\varepsilon}_{\text{ML}} = -\frac{1}{2\pi} \angle \gamma(\hat{\theta}_{\text{ML}})$$

Notice that only two quantities affect the log-likelihood function (and thus the performance of the estimator): the number of samples in the cyclic prefix L and the correlation coefficient ρ given by the SNR. The former is known at the receiver, and the latter can be fixed. Basically, the quantity $\gamma(m)$ provides the estimates of θ and ε . Its magnitude, which is compensated by an energy term, peaks at time instant $\hat{\theta}_{ML}$,

while its phase at this time instant is proportional to $\hat{\varepsilon}_{ML}$. If is *a priori* known to be zero, the log-likelihood function for becomes $\Lambda(\theta) = \text{re}\{\gamma(\theta)\} - \rho \phi(\theta)$, and $\hat{\theta}_{ML}$ is its maximizing argument. This estimator and a low-complexity variant are analyzed in [9].

In an OFDM receiver, the quantity $\gamma(\theta)$, which is defined in (6), is calculated on-line. The signals $\Lambda\left(\theta, \hat{\varepsilon}_{ML}(\theta)\right)$ (whose maximizing arguments are the time estimates $\hat{\theta}_{ML}$ and (whose values at the time instants yield the frequency estimates). Notice that (12) and (13) describe an open-loop structure. Closed-loop implementations based in (5) and (11) may also be considered. In such structures, the signal $\Lambda\left(\theta, \hat{\varepsilon}_{ML}(\theta)\right)$ is typically fed back in a phase-locked loop (PLL). If we can assume that θ is constant over a certain period, the integration in the PLL can significantly improve the performance of the estimators.

5. SIMULATION RESULTS

5.1 WORKING OF THE OFDM MODEL:

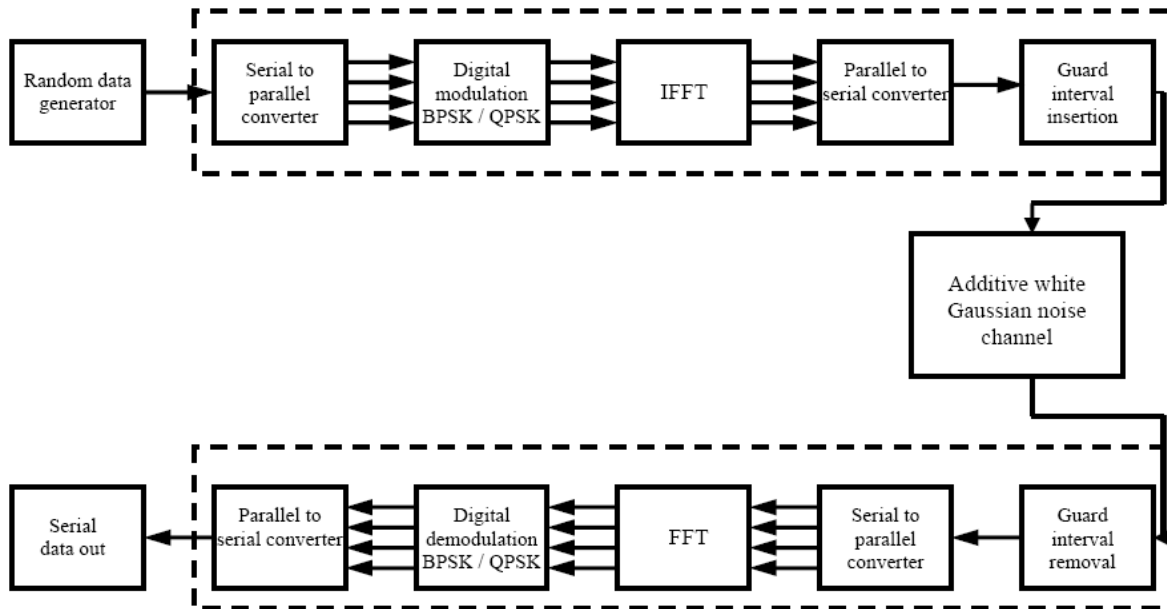


Fig 5.1 OFDM model used for simulation

Random data generator Serial to parallel converter Digital modulation BPSK / QPSK IFFT
 Parallel to serial converter Guard interval insertion Additive white Gaussian noise channel Guard
 interval removal Serial to parallel converter Parallel to serial converter Digital demodulation
 BPSK / QPSK Serial data out FFT Fig 6.1 OFDM model used for simulation

Figure 6.1 shows the basic block diagram of OFDM transmitter and receiver used for simulation. OFDM is generated by choosing the spectrum required, based on the input data, and modulation scheme used. Each carrier to be produced is assigned data to be transmitted. The required amplitude and phase of the carrier is then calculated based on the modulation scheme (typically BPSK, QPSK, or QAM). For example, if we have to transmit incoming 8 bit digital data, we have to choose 8 different carrier signals, which are orthogonal to each other. Each carrier is assigned to a different bit and its amplitude and phase are chosen according to modulation

scheme used. The required spectrum is then converted back to its time domain signal using an Inverse Fourier Transform.

In most applications, an Inverse Fast Fourier Transform (IFFT) is used [6, 7]. They are already in time domain, but here we pretend that the input bits are not time domain representations but are frequency amplitudes which if you are thinking clearly, will see that that is what they are. In this way, we can take these bits and by using the IFFT, we can create an output signal which is actually a time-domain OFDM signal. The IFFT is a mathematical concept and does not really care what goes in and what goes out. As long as what goes in is amplitudes of some sinusoids, the IFFT will crunch these numbers to produce a correct time domain result. Both FFT and IFFT will produce identical results on the same input. We insist that only spectrums go inside the IFFT. IFFT quickly computes the time-domain signal instead of having to do it one carrier at time and then adding. Calling this functionality IFFT may be more satisfying because we are producing a time domain signal, but it is also very confusing. Because FFT and IFFT are linear processes and completely reversible, it should be called a FFT instead of a IFFT. The results are the same whether you do FFT or IFFT. In literature you will see it listed as IFFT everywhere. This block can also be a FFT as long as on the receive side, you do the reverse. The IFFT performs the transformation very efficiently, and provides a simple way of ensuring the carrier signals proceed are orthogonal. The reverse process guarantees that the carriers generated are orthogonal.

Consider the model shown in Fig 6.1. The random data generator generates the data system. This input serial data stream is formatted into the word size required for transmission. For example, 1 bit/word for BPSK & 2 bits/word for QPSK and then shifted into a parallel format. The data is then transmitted in parallel by assigning each data word to one carrier in the transmission. The data to be transmitted on each carrier is then mapped into a Phase Shift Keying (PSK) format. The data on each symbol is mapped to a phase angle based on the modulation method. For example, in QPSK the phase angles used are 0° , 90° , 180° , and 270° . The use of phase shift keying produces a constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading.

After the required spectrum is worked out, an Inverse Fourier Transform is used to find the corresponding time domain waveform. The guard period is then added to the start of each symbol as shown in the fig 5.2.

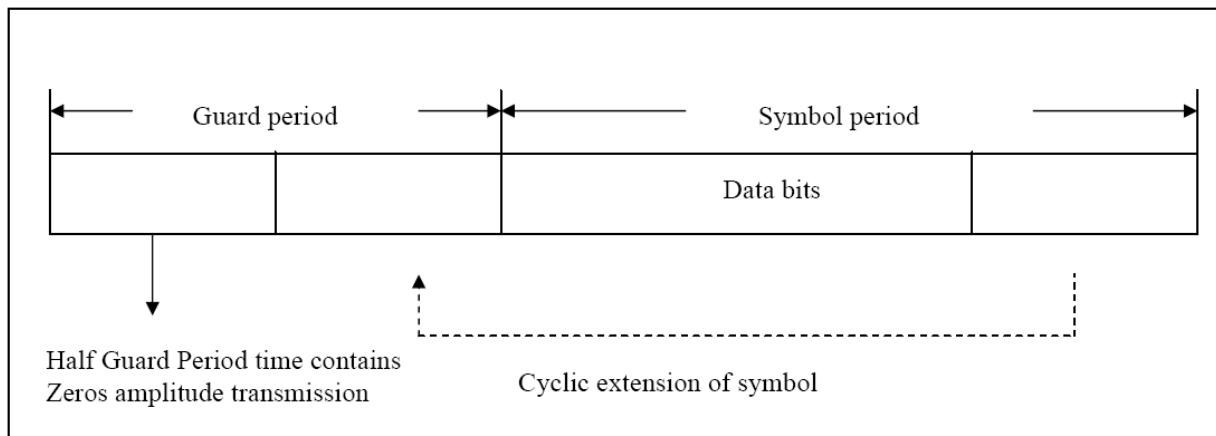
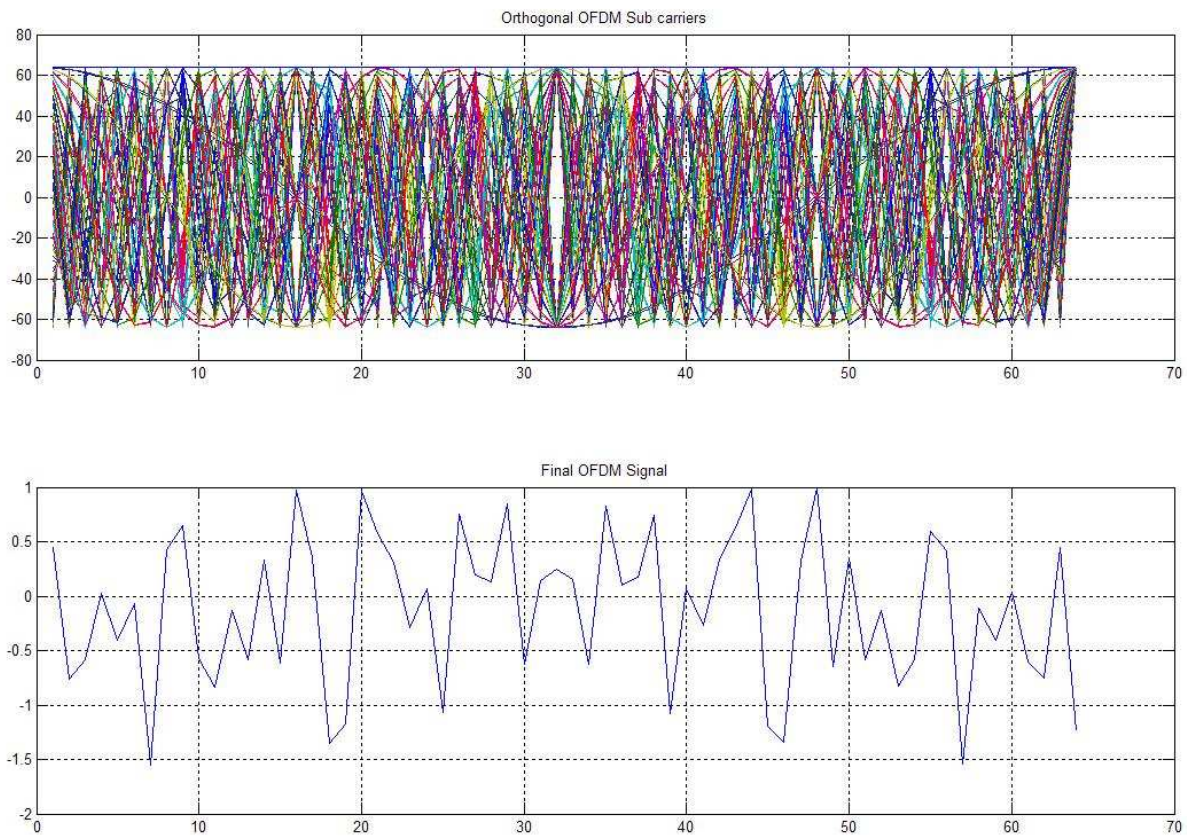


Fig 5.2 addition of guard band interval with symbol

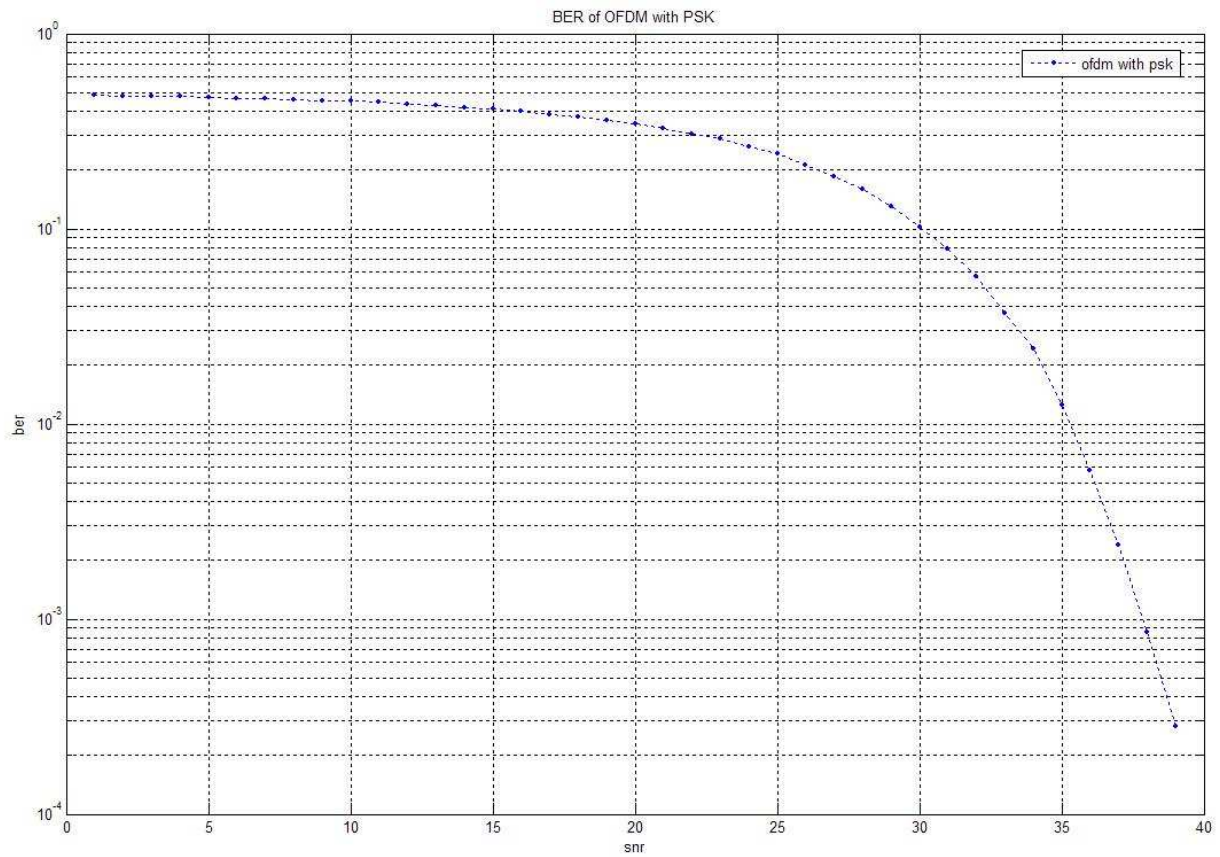
One of the most important properties of OFDM transmissions is its high level of robustness against multipath delay spread. This is a result of the long symbol period used, which minimizes the inter-symbol interference. The level of multipath robustness can be further increased by the addition of a guard period between transmitted symbols. The guard period allows time for multipath signals from the previous symbol to die away before the information from the current symbol is gathered. The guard period used, can be made up of two sections. Half of the guard period time is a zero amplitude transmission called Zero padding and the other half of the guard period is a cyclic extension of the symbol to be transmitted. After the guard has been added, the symbols are then converted back to a serial time waveform. This is then the base band signal for the OFDM transmission. A channel model is then applied to the transmitted signal. The model allows for the signal to noise ratio. It is set by adding a known amount of white noise to the transmitted signal. The channel output is given to the receiver. The receiver basically does the reverse operation to the transmitter. The guard period is removed from the received signal. The FFT of each symbol is then taken to find the original transmitted spectrum. The phase angle of each transmission carrier is then evaluated and converted back to the data word by demodulating the received phase. The data words are then combined, which gives the same word size as that of original data.

Simulation results:

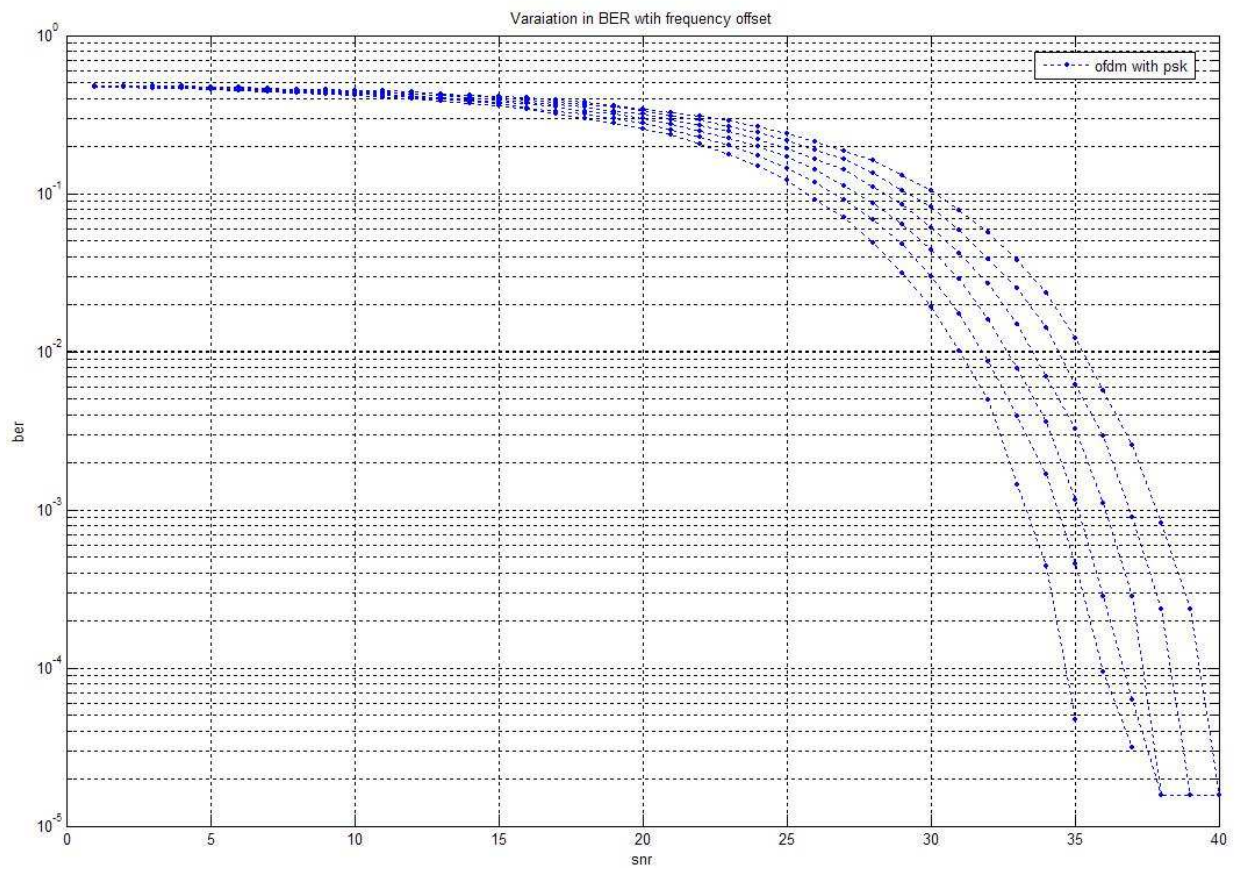
1. OFDM with 64 sub channels, simple OFDM symbol



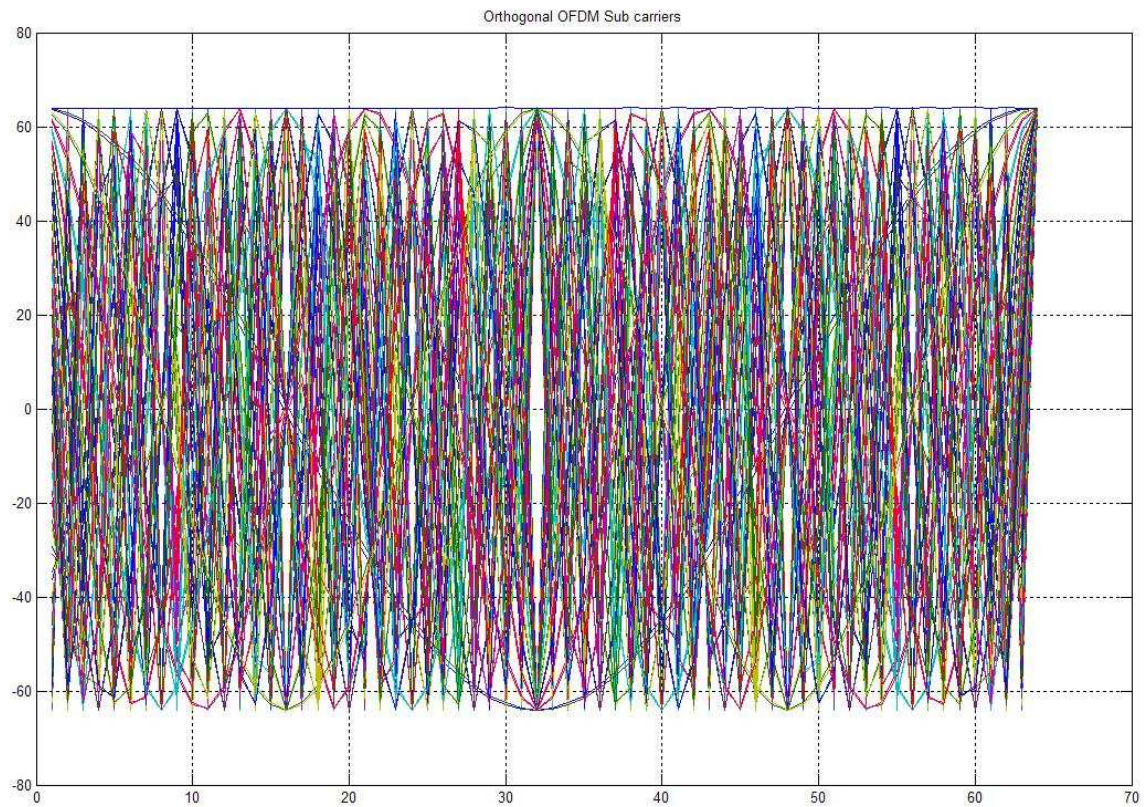
2. Bit error rate (BER) in OFDM using PSK as an Input Signal.



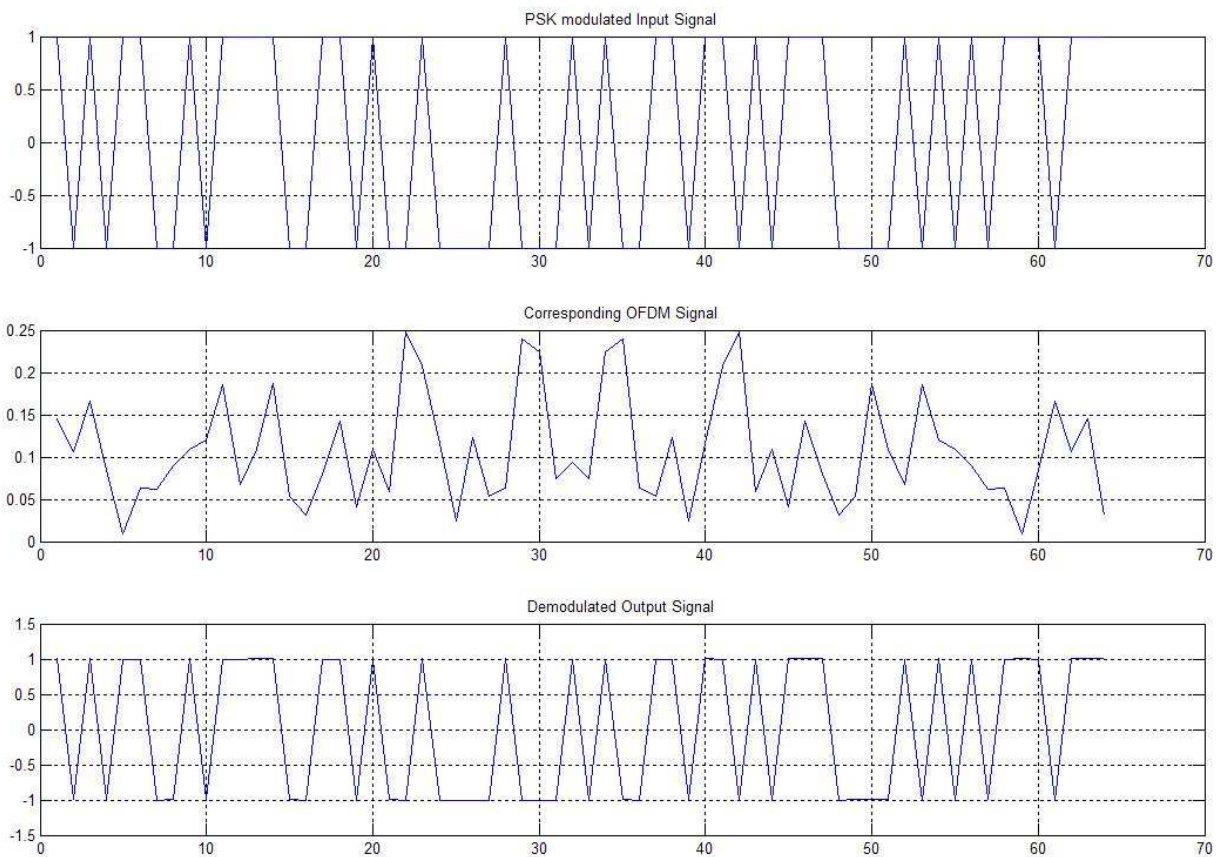
3. Effect of change in frequency offset in BER in OFDM.



4. Orthogonal subcarriers in an OFDM signal. $N=64$.



5. A Simple OFDM Signal with PSK modulated input signal ,and in absence of noise.



6. CONCLUSION

This chapter gives a summary of the work presented in this thesis. An outline for the future work based on this is also given.

6.1 SUMMARY

The OFDM makes efficient use of available spectrum by allowing overlapping among the carriers. It basically converts the high data rate stream in to several parallel lower data rate streams and thereby eliminating the frequency selective fading. It has been seen that the OFDM is a powerful modulation technique that is capable of high data rate and is able to eliminate ISI. It is computationally efficient due to the use of FFT techniques to implement modulation and demodulation functions.

Using MATLAB software, the performance of OFDM system was tested for two digital modulation techniques namely BPSK and QPSK.

From the simulation results, it is observed that the BPSK allows the BER to be improved in a noisy channel at the cost of maximum data transmission capacity. Use of QPSK allows higher transmission capacity, but at the cost of slight increase in the probability of error. This is because of the fact that QPSK uses two bits per symbol. Hence QPSK is easily affected by the noise. Therefore OFDM with QPSK requires larger transmit power. From the results, use of OFDM with QPSK is beneficial for short distance transmission link, whereas for long distance transmission link OFDM with BPSK will be preferable.

Maximum likelihood estimation method was implemented for the calculation of timing and frequency offsets. These frequency offsets are found to disturb the orthogonality of the OFDM symbols. And it was observed that using this ML estimation method we can improve the performance of any OFDM system. There are several other techniques also to predict the timing and frequency offsets introduced by the system.

6.2 SCOPE OF FUTURE WORK

The following are the some of the interesting extensions of the present work:

- 1) An interesting topic for future research is to perform more extensive performance comparisons between FFT based OFDM, DHT based OFDM, and DCT based OFDM systems under additional real-world channel impairments, such as multipath fading, time dispersion which leads to inter symbol interference (ISI).
- 2) The main problems with OFDM signal is very sensitive to carrier frequency offset, and its high Peak to Average Power Ratio (PAPR). So, these three transform based OFDM systems can be tested for these problems.

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